A Correspondence Principle for Scission-Induced Stress Relaxation in Elastomeric Components

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A method is presented for calculating the stress relaxation due to scission in elastomeric components that operate at a fixed deformation while at an elevated temperature. A relationship is established between stresses at different temperatures that is called the correspondence principle for scission/healing materials. Two examples involving cylinders illustrate its use. The first example involves combined tension-torsion, for which an axial force-twisting moment relation is derived, that might be useful in experimental studies to assess the applicability of the correspondence principle. The second example provides a criterion for estimating the lifetime of an annular seal.
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1 Introduction

In applications where the mechanical and thermal loads on elastomeric structural components are benign enough that no changes in microstructure occur, stresses and deformations can be calculated using the nonlinear theory of elasticity. However, when the temperature (or deformation) of an elastomeric structural component is sufficiently large, scission of molecular cross-links and possible in situ recross-linking (healing) can result in significant time-dependent softening of mechanical properties as well as permanent set [1,2]. In applications involving elastomeric seals or bushings in automotive or truck suspension systems, for example, these changes can impair performance and require eventual replacement. In such applications, where it is important to be able to predict the lifetime of the elastomeric component, the nonlinear theory of elasticity is no longer applicable, and a new means for determining stresses is required.

In this paper, a method is presented which can be used to calculate the stress relaxation due to scission in elastomeric components that operate at a fixed deformation while at elevated temperatures as, for example, could occur in seals or bushings. The method is based on a correspondence that is established between stresses in an elastomeric component at different temperatures. Because an analogous situation in the linear theory of viscoelasticity has proven to be very useful [3], the method presented here is referred to as the *correspondence principle for scission/healing materials*.

The proposed *correspondence principle* is restricted to conditions when the deformation is fixed and the temperature is spatially uniform. There are applications when these conditions should be at least approximately satisfied. For example, if the surface temperature of a seal is increased, the time required for the temperature field within the seal to become uniform may be small compared to the time for there to be significant scission. Also, a seal may be subjected to a large initial deformation and then small superposed deformations as, for example, would occur if the seal were used in a vibrating pump. Thus, the conditions for the application of the *correspondence principle* would be at least approximately satisfied and would lead to a useful first approximation for the relaxation of stresses in an elastomeric component undergoing scission.

The constitutive theory that accounts for scission-induced stress relaxation in an elastomer at an arbitrary fixed deformation is presented in Sec. 2. The *correspondence principle for scission/healing materials* is developed in Sec. 3. Two examples involving cylinders are presented in Sec. 4. The first involves combined tension-torsion. A result relating the axial force and twisting moment is derived which might be useful in experimental studies. The second example provides a criterion for estimating the lifetime of a seal.

2 Constitutive Equation

Tobolsky [1] described experiments in which a rubber strip at room temperature was subjected to a fixed uniaxial stretch and then held at a higher fixed temperature for a specified time interval. At temperatures above T_{cr} (say 100 °C), called the chemorheological temperature, the stress decreased with time. At the end of the specified time interval, the external force was reduced to zero and the specimen was returned to its original temperature. The specimen was observed to have a permanent stretch. Tests were carried out for different applied stretches, temperatures, and time intervals. The decrease in tensile stress with time and the permanent stretch were measured. Results of more recent experiments can be found in the article by Wineman, Jones, and Shaw [2]. Tobolsky analyzed the data assuming the elastomer to be instantaneously neo-Hookean, for which the relation between tensile (Cauchy) stress $\sigma(t)$ and uniaxial stretch ratio λ is

$$\sigma(t)=2n(t)kT\left(\lambda^2-\frac{1}{\lambda}\right) \tag{1}$$

where T is the absolute temperature, k is the Boltzmann constant, and $n(t)$ is the current cross-link density. The decrease in $\sigma(t)$ was attributed to scission of molecular network cross-links, resulting in a decrease in $n(t)$. The permanent stretch was attributed to a new network that formed in the stretched state (healing). At temperatures below T_{cr}, the stress-stretch relation for the system consisting of the two networks was assumed to be

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$$\sigma = 2n_1 kT\left(\lambda^2 - \frac{1}{\lambda}\right) + 2n_2 kT\left[\left(\frac{\lambda}{\hat{\lambda}}\right)^2 - \left(\frac{\hat{\lambda}}{\lambda}\right)\right] \tag{2}$$

where $\hat{\lambda}$ is the stretch ratio of the original network while held at the high temperature, n_1 is the cross-link density of the original network at the end of the test, and n_2 is the cross-link density of the new network. Equation (2) expresses the assumptions that i) the total stress is the sum of the stresses in each network, ii) each network acts as an incompressible isotropic neo-Hookean elastic material, and iii) broken cross-links reform to produce a new network that is stress free when the stretch ratio of the original network is $\hat{\lambda}$ [1,4].

Tobolsky's data suggested that $n(t)$ in (1) is independent of the stretch ratio $\hat{\lambda}$ up to a value of about 4. This was supported by the results of Scanlan and Watson [5]. According to Eq. (1), $\sigma(t)/\sigma(0) = \beta(T,t)$ and $n(t) = \beta(T,t)n_o$ where n_o is the initial cross-link density. $\beta(T,t)$ is a material property function that can be obtained experimentally (see [1], Fig. V.4). Tobolsky ([1], p. 226) suggested that $\beta(T,t)$ can be represented in the form

$$\beta(T,t) = \phi(\alpha(T)t) \tag{3}$$

for a number of elastomers. For a particular natural rubber vulcanizate in the temperature range $100\,°\mathrm{C} \leq T \leq 130\,°\mathrm{C}$, Tobolsky showed that

$$\beta(T,t) = \exp(-\alpha(T)t), \tag{4}$$

with

$$\alpha(T) = \frac{k_B}{k_P} T \exp(-E_{act}/RT) \tag{5}$$

k_B is Boltzmann's constant, k_P is Planck's constant, E_{act} is an activation energy whose value was found to be 30.4 kcal/mol, and R is the gas constant.

Neubert and Saunders [6] carried out tests similar to those of Tobolsky, but for a pure shear deformation. They measured permanent biaxial stretch upon removal of stress and reduction of the temperature to its original value, and found that predictions based on the assumption of a neo-Hookean response led to inaccurate predictions of permanent set. They modified assumption ii) by modeling the rubber as a Mooney-Rivlin material, and showed that this model led to better agreement with the measured permanent biaxial stretch. Fong and Zapas [7] later proposed using the Rivlin-Saunders model to determine the permanent biaxial stretch.

These results are now used as a guide for the development of a constitutive framework for the three-dimensional response of a rubber undergoing scission while at a fixed homogeneous deformation and constant temperature history. For a detailed discussion of the constitutive equation, see Wineman and Shaw [8]. Consider a rubbery material in a stress-free reference configuration at a temperature T. There is a range of deformations and temperatures for which the material response can be regarded as incompressible, isotropic, and nonlinearly elastic. If $\mathbf{x}$ is the position at current time t of a particle located at $\mathbf{X}$ in the reference configuration, the deformation gradient is $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$. The left Cauchy-Green tensor is $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ and the Cauchy stress $\boldsymbol{\sigma}$ is given by

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1} \tag{6}$$

where p is an arbitrary hydrostatic pressure arising from the constraint that deformations are isochoric. I_1, I_2 are the first and second invariants of $\mathbf{B}$, respectively, and $W(I_1, I_2, T)$ is the strain energy density associated with the original material. In Eq. (6), $\boldsymbol{\sigma}, \mathbf{B}$, and T are evaluated at the current time t, which is omitted from the notation for brevity. For many proposed models of rubber elasticity, the strain energy density function is written as $W(I_1, I_2, T) = n_o kTW^o(I_1, I_2)$, that is, the dependence on temperature and deformation is separable. Note that the subscript or superscript o indicates the scission-independent part of a quantity. This is the case for the phantom model, affine model, constrained chain model, localization model, liquidlike model, and eight-chain model [9]. Accordingly, Eq. (6) can be restated as

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\sigma}^o(\mathbf{B}, T) \tag{7a}$$

where

$$\boldsymbol{\sigma}^o(\mathbf{B}, T) = 2n_o kT\mathbf{S}^o(\mathbf{B}), \quad \mathbf{S}^o(\mathbf{B}) = \frac{\partial W^o}{\partial I_1}\mathbf{B} - \frac{\partial W^o}{\partial I_2}\mathbf{B}^{-1}. \tag{7b}$$

For temperatures $T < T_{cr}$ and moderate deformations, no microstructural changes are assumed to occur, and the stress is given by Eq. (6), $(7a)$, or $(7b)$. If the material is held at a fixed homogeneous deformation and the temperature is increased to a fixed value $T \geq T_{cr}$ at time $t = 0$, scission of the original microstructural network is assumed to occur continuously in time. The volume fraction of the original network cross-link density at time t is denoted as $\beta(T,t)$, a monotonically decreasing function of t satisfying $\beta(T,0) = 1$.

The current stress is given by

$$\boldsymbol{\sigma} = -p\mathbf{I} + \beta\left[2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1}\right], \tag{8a}$$

or, alternatively,

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\beta n_o kT\left[\frac{\partial W^o}{\partial I_1}\mathbf{B} - \frac{\partial W^o}{\partial I_2}\mathbf{B}^{-1}\right], \tag{8b}$$

where β and $\boldsymbol{\sigma}$ are evaluated at the current time t. This constitutive equation extends the ideas inherent in Eq. (1) to arbitrary homogeneous deformations. Neubert and Saunders [6] used it in their analysis of the permanent set due to new networks that formed during a pure shear deformation.

Several comments are in order regarding constitutive Eq. (8). First, in accordance with Tobolsky's experimental results, $\beta(T,t)$ is assumed to be independent of the deformation. This is strictly justified only for fixed uniaxial extensions with $\lambda < 4$. There is a lack of experimental evidence for other deformations. Second, although Tobolsky assumed the response of the original and newly formed networks to be neo-Hookean, Neubert, and Saunders [6] and Fong and Zapas [7] considered other possibilities. Thus, $W^o(I_1, I_2)$ is left unspecified. Third, consistent with assumption iii) above, new networks that result from cross-linking are formed in a stress-free state. Provided the deformation is held fixed, these new networks do not contribute to the stress and no further constitutive assumptions are required.

3 Correspondence Principle for Scission-Healing Materials

Consider an elastomeric body that has been subjected to a nonhomogeneous deformation and is in equilibrium at a spatially uniform temperature $T_o < T_{cr}$. Let its deformed configuration be denoted by κ. Surface tractions are specified on the portion of the deformed surface denoted as $\partial\kappa^{(\sigma)}$ and the current positions of particles are specified on the portion of the deformed surface denoted as $\partial\kappa^{(d)}$. Let $\hat{\mathbf{x}}$ denote the prescribed current particle positions on $\partial\kappa^{(d)}$, $\hat{\mathbf{T}}^{(\sigma)}$ denote the prescribed surface traction on $\partial\kappa^{(\sigma)}$, and $\hat{\mathbf{T}}^{(d)}$ denote the computed surface traction on $\partial\kappa^{(d)}$. The stress and deformation fields satisfy the following conditions:

$$\operatorname{div}\boldsymbol{\sigma} = \mathbf{0} \quad \text{in} \quad \kappa, \tag{9a}$$

$$\boldsymbol{\sigma}\mathbf{n} = \hat{\mathbf{T}}^{(\sigma)} \quad \text{on} \quad \partial\kappa^{(\sigma)}, \tag{9b}$$

$$\mathbf{x} = \hat{\mathbf{x}} \quad \text{on} \quad \partial\kappa^{(d)} \tag{9c}$$

where $\mathbf{n}$ denotes the unit outer normal at a point of the external surface. The constitutive equation is given by Eqs. $(7a)$ and $(7b)$, which when substituted into Equation $(9a)$, gives

$$-\text{grad}\, p_o + 2n_o kT_o \,\text{div}\, \mathbf{S}^o = \mathbf{0} \tag{10}$$

Boundary condition $(9b)$ with $(7b)$ can be written in the form

$$-p_o\mathbf{n} + 2n_o kT_o \mathbf{S}^o \mathbf{n} = \hat{\mathbf{T}}^{(\sigma)} \tag{11}$$

Equations (9*c*), (10), and (11) define a boundary value problem for the scalar field $p_o(\mathbf{X})$ and deformation $\mathbf{x}_o(\mathbf{X})$. The corresponding stresses are given by

$$\boldsymbol{\sigma}_o = -p_o \mathbf{I} + 2n_o kT_o \mathbf{S}^o(\mathbf{B}_o) \tag{12}$$

where $\mathbf{B}_o$ is calculated from $\mathbf{x}_o(\mathbf{X})$. The surface tractions $\hat{\mathbf{T}}^{(d)}$ on $\partial\kappa^{(d)}$ are

$$\hat{\mathbf{T}}^{(d)} = \boldsymbol{\sigma}_o \mathbf{n} = -p_o\mathbf{n} + 2n_o kT_o \mathbf{S}^o(\mathbf{B}_o)\mathbf{n} \tag{13}$$

Now, suppose the body is brought to a higher, spatially uniform temperature $T_1 > T_{cr}$ and is in equilibrium at a fixed deformation. According to various researchers (e.g., [1,5]), the volume changes associated with the temperature change of interest and the process of scission and subsequent reforming of cross-links are small enough to be neglected. It is thus assumed that the body has the same deformation as when at the lower temperature $T_o < T_{cr}$, that is $\mathbf{x} = \mathbf{x}_o(\mathbf{X})$. Constitutive Eq. $(8b)$ gives

$$\boldsymbol{\sigma}_1 = -p_1 \mathbf{I} + 2\beta(T_1,t) n_o kT_1 \mathbf{S}^o(\mathbf{B}_o) \tag{14}$$

Equilibrium condition $(9a)$ becomes, using Eq. (14),

$$\text{div}\, \boldsymbol{\sigma}_1 = -\text{grad}\, p_1 + 2\beta(T_1,t) n_o kT_1 \,\text{div}\, \mathbf{S}^o(\mathbf{B}_o) = \mathbf{0} \tag{15}$$

Since $p_o(\mathbf{X})$ and $\mathbf{x}_o(\mathbf{X})$ satisfy Eq. (10), Eq. (15) becomes

$$\begin{aligned} \text{div}\, \boldsymbol{\sigma}_1 &= -\text{grad}\, p_1 + \beta(T_1,t) T_1/T_o \,\text{grad}\, p_o \\ &= \text{grad}(-p_1 + p_o \beta(T_1,t) T_1/T_o) = \mathbf{0} \end{aligned} \tag{16}$$

If we let $p_1 = p_o(\mathbf{X})\beta(T_1,t)/T_1/T_o$, the equilibrium equation is satisfied.

The corresponding stress is found from Eq. (14),

$$\boldsymbol{\sigma}_1 = \beta(T_1,t) T_1/T_o[-p_o \mathbf{I} + 2n_o kT_o \mathbf{S}^o(\mathbf{B}_o)] \tag{17}$$

Boundary condition $(9c)$ is automatically satisfied because of the assumed deformation. The tractions on the deformed external surface are calculated using Eq. (17),

$$\boldsymbol{\sigma}_1 \mathbf{n} = \beta(T_1,t) T_1/T_o[-p_o \mathbf{n} + 2n_o kT_o \mathbf{S}^o(\mathbf{B}_o)\mathbf{n}] \tag{18}$$

Evaluating Eq. (18) on $\partial\kappa^{(\sigma)}$ gives $\beta(T_1,t)T_1/T_o\hat{\mathbf{T}}^{(\sigma)}$ and on $\partial\kappa^{(d)}$ gives $\beta(T_1,t)T_1/T_o\hat{\mathbf{T}}^{(d)}$.

The results of this section establish the following:

Correspondence Principle for Scission/Healing Materials

Let $\mathbf{x} = \mathbf{x}_o(\mathbf{X})$ be an equilibrium deformation for an incompressible isotropic elastomeric body at a spatially uniform temperature $T_o < T_{cr}$ and let the corresponding stress field be denoted as $\boldsymbol{\sigma}_o(\mathbf{X})$. Then $\mathbf{x} = \mathbf{x}_o(\mathbf{X})$ is also an equilibrium deformation when the body is brought to a higher, spatially uniform temperature $T_1 > T_{cr}$, where it undergoes the scission-recross-linking process. The corresponding stresses are $\boldsymbol{\sigma}_1(\mathbf{X},t) = \beta(T_1,t)T_1/T_o\boldsymbol{\sigma}_o(\mathbf{X})$, where $\beta(T_1,t)$ is the material's scission response function, i.e., the ratio of the current to the original cross-link density for the original material. If the surface tractions are $\hat{\mathbf{T}}^{(\sigma)}$ on $\partial\kappa^{(\sigma)}$ and $\hat{\mathbf{T}}^{(d)}$ on $\partial\kappa^{(d)}$ at $T_o < T_{cr}$, then at $T_1 > T_{cr}$ the surface tractions are $\beta(T_1,t)T_1/T_o\hat{\mathbf{T}}^{(\sigma)}$ and $\beta(T_1,t)T_1/T_o\hat{\mathbf{T}}^{(d)}$, respectively.

4 Applications of the Correspondence Principle for Scission/Healing Materials

In this section, two examples are presented to examine the consequences of the *correspondence principle for scission/healing materials*. The first example discusses a nontrivial multiaxial deformation state that could lead to a nice experimental assessment of the validity of the proposed correspondence principle. The second example is a practical application of the correspondence principle, showing a method for determining the lifetime of an elastomeric seal at an elevated temperature.

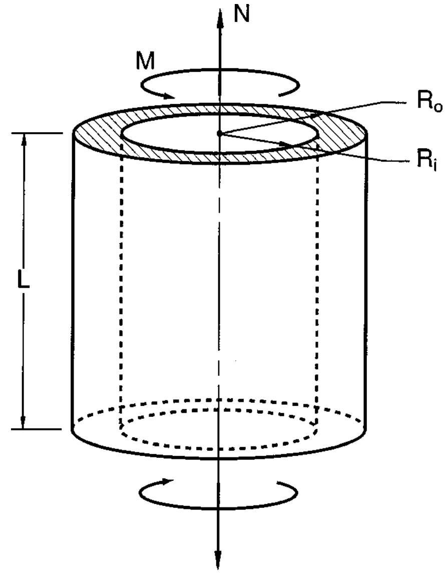

Fig. 1 Hollow cylinder subjected to axial force and torsion

Combined Tension-Torsion of a Circular Cylinder. Consider a circular cylinder at a uniform temperature $T_o < T_{cr}$ with length L, inner radius R_i, and outer radius R_o. The inner and outer cylindrical surfaces are traction free and axial force N and twisting moment M are applied to its end surfaces (see Fig. 1). The cylinder is in equilibrium under these applied loads.

The resulting deformation is assumed to be axially symmetric, in which plane cross sections remain plane, displace along and rotate about the axis of symmetry, and cylindrical surfaces deform into cylindrical surfaces. Let a cylindrical coordinate system be introduced that is coaxial with the cylinder and has its origin at one end. A material point at (R,Θ,Z) in the reference configuration deforms to (r,Θ,z) in the current configuration. The mapping describing this deformation has the form

$$\begin{aligned} r &= \left[\frac{1}{\lambda} R^2 + \gamma\right]^{1/2} \\ \theta &= \Theta + \psi\lambda Z \\ z &= \lambda Z \end{aligned} \tag{19}$$

λ and ψ are constants that represent the uniform axial stretch ratio and uniform cross-sectional rotation per current length, respectively. If r_i and r_o are the radii of the deformed inner and outer surfaces, then

$$\frac{1}{\lambda} = \frac{r_o^2 - r_i^2}{R_o^2 - R_i^2}, \qquad \gamma = \frac{R_o^2 r_i^2 - R_i^2 r_o^2}{R_o^2 - R_i^2} \tag{20}$$

Consider a possible experiment in which λ and ψ are specified and the cylindrical surfaces are traction free in the current configuration. Using the notation of Sec. 3, the inner and outer cylindrical surfaces form $\partial\kappa^{(\sigma)}$ and $\hat{\mathbf{T}}^{(\sigma)} = \mathbf{0}$. The ends of the cylinder, $z=0$ and $z=\lambda L$, form the surface $\partial\kappa^{(d)}$. $\hat{\mathbf{x}}$ is obtained by evaluating the mapping in Eq. (19) at $Z=0$ and $Z=L$.

An analysis of the combined torsion and tension of a circular cylinder can be found in [10]. A scalar field $p_o(r)$ can be found so that the equilibrium equation is met. The radii r_i and r_o of the deformed cylindrical surfaces are determined from the first of Eq. (20) and the boundary condition that $\hat{\mathbf{T}}^{(\sigma)} = \mathbf{0}$ on $\partial\kappa^{(\sigma)}$. The constant γ is then known. Expressions for the stress are presented in

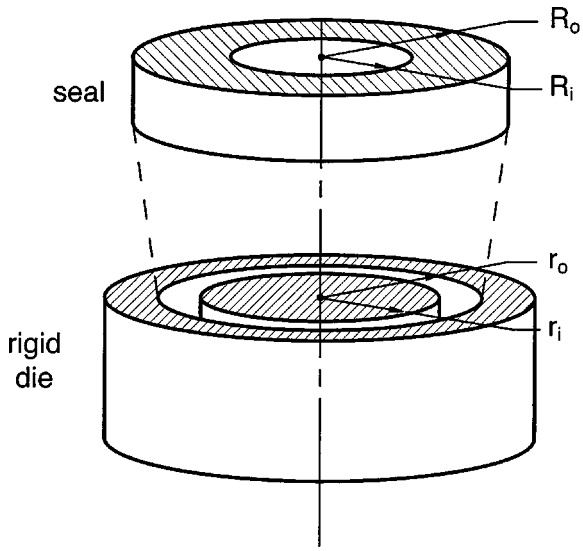

Fig. 2 Elastomeric seal and rigid die

[9] but are omitted here for brevity. Of particular interest here are the expressions for the axial force and twisting moment applied the ends of the cylinder,

$$N=2\pi\int_{r_i}^{r_o}\sigma_{zz}r\,dr \tag{21a}$$

$$M=2\pi\int_{r_i}^{r_o}\sigma_{z\theta}r^2\,dr \tag{21b}$$

Suppose that the temperature of the cylinder is increased to $T_1>T_{cr}$. According to the correspondence principle, the deformation $\mathbf{x}=\mathbf{x}_o(\mathbf{X})$ is given by Eq. (19). The traction on $\partial\kappa^{(\sigma)}$ is $\beta(T_1,t)T_1/T_o\hat{\mathbf{T}}^{(\sigma)}=\mathbf{0}$. On $\partial\kappa^{(d)}$, $\sigma_{zz}|_{T_1}=\beta(T_1,t)T_1/T_o\sigma_{zz}|_{T_0}$, and $\sigma_{z\theta}|_{T_1}=\beta(T_1,t)T_1/T_o\sigma_{z\theta}|_{T_o}$. Since Eq. (21) applies at all temperatures, it follows that

$$N|_{T_1}=2\pi\int_{r_i}^{r_o}\sigma_{zz}|_{T_1}r\,dr=2\pi\int_{r_i}^{r_o}\beta(T_1,t)T_1/T_o\sigma_{zz}|_{T_1}r\,dr$$
$$=\beta(T_1,t)T_1/T_oN|_{T_o} \tag{22a}$$

In a similar manner it can be shown that

$$M|_{T_1}=\beta(T_1,t)T_1/T_oM|_{T_o} \tag{22b}$$

It also follows that

$$\frac{N|_{T_1}}{M|_{T_1}}=\frac{N|_{T_o}}{M|_{T_o}} \tag{23}$$

is a constant, independent of time. Thus, the axial force and twisting moment have the same decrease with time. This is a result that could potentially be assessed experimentally by measuring the time dependence of the normal force and twisting moment at high temperatures to determine the validity of the proposed correspondence principle.

Lifetime Prediction for Elastomeric Seals. Consider the cylinder of the previous example once again at a uniform temperature $T_o<T_{cr}$. Let it now be force fit and thus seal the annular space between an inner rigid cylindrical of radius $r_i>R_i$ and outer rigid cylinder of radius $r_o<R_o$. Let the ends of the cylinder be traction free (see Fig. 2).

The resulting deformation is assumed to be axially symmetric. The end surfaces form the portion of the boundary $\partial\kappa^{(\sigma)}$ and the cylindrical surfaces form the portion of the boundary $\partial\kappa^{(d)}$. Suppose it is assumed that the deformation is given by Eq. (19) with $\psi=0$. It can then be shown that $M=0$. Since r_i and r_o are specified, λ and γ are determined from Eq. (20). In general, the boundary condition $\hat{\mathbf{T}}^{(\sigma)}=\mathbf{0}$ cannot be satisfied at each point on $\partial\kappa^{(\sigma)}$. Instead, let the relaxed boundary condition $\mathbf{N}=0$ be imposed. The scalar field $p_o(r)$ and the stress components are then completely determined. σ_{rr}, the radial stress, is the only nonzero stress component acting on $\partial\kappa^{(d)}$, and the pressure between the seal and a rigid cylinder is $|\sigma_{rr}|$. Suppose that the pressure between the seal and a cylindrical surface must be at least p^* if a leak is to be avoided. When $T_o<T_{cr}$, r_i and r_o can be chosen so that $|\sigma_{rr}(r_o)|_{T_o}>p^*$ and $|\sigma_{rr}(r_i)|_{T_o}>p^*$.

Suppose that the temperature of the seal is increased to $T_1>T_{cr}$ at $t=0$. According to the correspondence principle, the deformation is unchanged. Using the discussion from the first example, it can be shown that $N|_{T_1}=0$. The pressures for $t>0$ between the seal and the inner and outer rigid cylinders are, respectively,

$$p_{inner}(t)=|\sigma_{rr}(r_i)|_{T_1}=\beta(T_1,t)T_1/T_o|\sigma_{rr}(r_i)|_{T_o},$$
$$p_{outer}(t)=|\sigma_{rr}(r_o)|_{T_1}=\beta(T_1,t)T_1/T_o|\sigma_{rr}(r_o)|_{T_o}. \tag{24}$$

Because of scission, these pressures will relax with time. Leakage is predicted to occur at the smallest time t^* when

$$\min\{p_{inner}(t^*),p_{outer}(t^*)\}=p^* \tag{25}$$

is reached, thereby giving an estimate of the seal's useful life at temperature T_1.

5 Summary and Conclusions

A correspondence principle has been introduced which can be used to calculate the stress relaxation due to scission in an elastomeric component at an elevated temperature T_1 in terms of the stresses in the component at a lower temperature T_o where there is no scission. The application of the principle assumes knowledge of two items: 1) the stress distribution at temperature T_o, determined by either analytical or numerical methods and 2) a material property $\beta(T,t)$ that can be determined from uniaxial stress relaxation experiments at different constant temperatures. The correspondence principle requires that the deformations be the same at T_1 and T_o and that the temperature fields be homogeneous. When these conditions are at least approximately satisfied, the correspondence principle can give a useful first approximation to the actual stresses during scission. Two examples illustrate the application of the correspondence principle. In the first, a tension-torsion experiment can be used to assess the validity of the principle. In the second example, the usable lifetime of a seal at high temperature can be predicted.

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